


# Methods for Quantifying Patterns of Dynamic Interactions in Dyads

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## Abstract

Individuals in both clinical and research settings increasingly provide data across numerous time points. Examples include measurements collected from wearable technology (e.g., accelerometers), psychophysiological measures, coded observations, social media behaviors, and daily diary data. When numerous observations are available for each individual, the data fall under the class of time series data and can be examined from within a dynamic systems perspective. We provide a broad overview of current analytic methods for quantifying relations among dyads using this data type. The techniques include those from within a linear modeling framework, approaches that include a measurement model, and methods for examining cyclical relations. We also discuss some special topics, such as methods that allow for models to shift across time and that accommodate heterogeneity across individuals. Finally, methods that account for similar shapes of nonlinear curves across time are described. From this breadth of options, we hope to help guide practitioners, clinicians, and researchers in choosing the optimal method for their data and line of questions. To further aid in this choice, we indicate programs available for each technique. Example dyads presented here range from mother–infant, patient–caretaker, and husband–wife; however, the analytic methods can be applied to any type of dyad.

## Keywords

analytic methods, time series analysis, dyadic interactions, dynamic system

Observing dynamic interactions among individuals has long been a practice for professionals working with families, couples, and in other settings where social processes are of interest. The possibility of quantifying aspects of dyadic interactions that may be of clinical interest continues to grow given the increase in ability to gather numerous observations across time for individuals. For instance, emerging methods can empirically identify the influence that one partner in a marriage exerts on the other's mood, or the coherence between a client's and a patient's recordings of heart rate. Sometimes referred to as "intensive longitudinal data," examples include data from daily diary questionnaires, ecological momentary assessments, and observational data. For generalizability to other types of data that may be used, such as psychophysiological data, we will refer to these data as "time series data" here. The present article provides an overview of statistical techniques for quantifying dyadic data from within a dynamic system framework. Please note that these methods can also be used on any multivariate time series data. The present article is pedagogical in nature, focusing only on approaches that are readily available.

Dynamic systems approaches provide the ideal mathematical tools for analyzing dyadic interactions. Broadly, a dynamic system is one that changes over time, with its

current state dependent on its previous states (Alligood, Sauer, & Yorke, 1996; Chow, Ferrer, & Nesselrode, 2007). An important benefit of studying dynamic interactions is the ability to examine how two individuals relate across time. Traditionally, researchers might collect data across time for a given individual and then average that individual's data. From this, researchers can look at the sample from a cross-sectional standpoint. This differs from the time series approaches to be outlined here. Prior work has demonstrated that features of the data aside from the average, such as the degree to which a person's mood is stable across days (Castro-Schilo & Ferrer, 2013), can reveal new insights. Similarly, the degree to which two individuals covary across time can elucidate the processes defining their interaction. The present article provides a guide for choosing a method that best matches the research question regarding dyadic interactions.

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**Table 1.** Methods for dyad-level modeling.

Method	Research questions answered	Recommendations and limitations	Implementation
Cross-corr.	Do the actors covary at a lag and/or contemporaneously?	Does not include third variable arguments or measurement error. Excellent for continuous data.	Standard statistics software
VAR	Do the actors covary across time, controlling for other variables?	Does not include measurement error. Excellent for continuous data.	Standard statistics software
SSM	Do the actors covary on a latent construct, controlling for other variables?	Excellent for continuous data.	SsfPack, (Koopman, Shephard, & Doornik, 1999)
uSEM	Do the actors covary at a lag and/or contemporaneously, controlling for other variables?	Estimates of weights are more biased than SSM estimation approaches. Excellent for continuous data.	R packages: <i>vars</i> (Pfaff, 2008); <i>gimme</i> (Lane, Gates, & Molenaar, 2015)
HMM	What describes the states the actors mutually share? How does this dyad move in and out of these states?	Only allows for one time lag. Excellent for categorical data.	R package: <i>depmix</i> (Visser, 2014)
GridWare	Is the dyad rigid or flexible in their shared behavior states? How long are they in each state?	Provides descriptive measures necessitating further analysis. Excellent for categorical data.	GridWare (Lamey, Hollenstein, Lewis, & Granic, 2004)
RQA	Is the dyad rigid or flexible in their shared behavior states? How long are they in each state?	Provides descriptive measures necessitating further analysis. Excellent for categorical data.	R package: CRQA (Coco & Dale, 2014)
Diff. eq.	Is change in one actor predicted by the other actor at a previous time, controlling for other variables?	Does not take into account contemporaneous effects. Excellent for categorical data.	SAS proc model
Freq. domain	Do the actors share similar cyclical patterns? Does one actor's cycle onset precede the other actor's?	Difficulties in interpretation. Excellent for continuous data.	Standard statistics software

Note. Cross-corr. = cross-correlation; VAR = vector autoregression; SSM = state-space modeling; uSEM = unified structural equation modeling; HMM = hidden Markov modeling; RQA = recurrence quantification analysis; Diff. eq. = differential equations; Freq. domain = frequency domain analysis.

## Methods

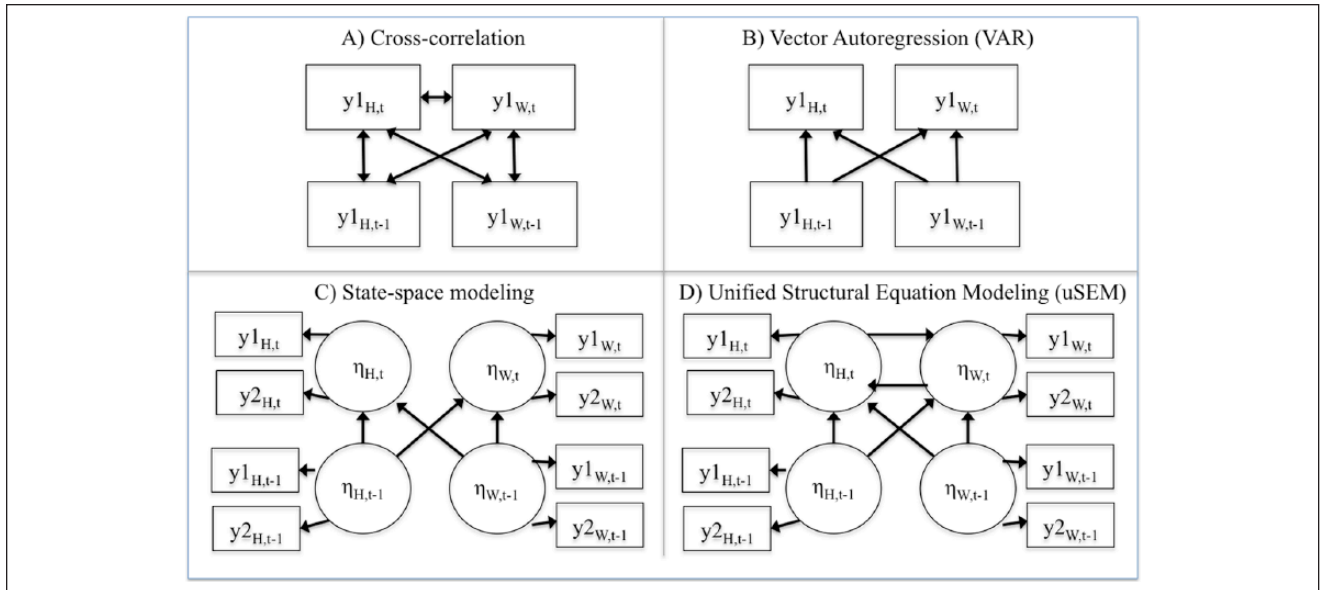
Most modeling approaches described below share a few properties unless otherwise specified. The first is that the methods require that the assumption of weak stationarity for the data be met. For time series data to have weak stationarity, its mean, variance, and covariance must remain constant across time (Shumway & Stoffer, 2006). That is, the features of the dyad's time series must be relatively consistent across time, with no jumps or shifts. Second, most approaches require continuous or scaled data as opposed to categorical data. Third, in terms of organizing the data, all the data analytic techniques described below require that the data be in long form. This means that there are  $T$  rows of data and  $p$  columns of variables, where  $T$  is the length of time (or observations) and  $p$  is the total number of variables to be used in analysis. For adequate power, the same suggestions used for many types of general linear models are followed.

Despite some similarities in the requirements, the methods vary greatly in terms of the features offered, questions they can attend to, how they have been used, and the ease with which it can be used by clinicians and researchers.

Additionally, some methods have different assumptions regarding the data. These points are described in full for each method. We organize models in terms of dyad-level models, or those that are conducted for each dyad (summarized in Table 1), and group-level models, or those that in some way use information across the sample of dyads (Table 2). Please note that for consistency in describing the methods, we refer to the dyad in terms of a husband-wife unit; however, the methods can be used for any dyad.

### Dyad-Level Methods

**Cross-Correlation-Based Methods.** Averaging values across time can be informative, but additional information may be reaped from these data. In particular it may be of interest to see how two individuals covary across time, or the extent to which variation in one individual relates to variation in another individual (see Figure 1A). The ideal data for this type of analysis are similar to data typically desired for any correlational analysis: continuous, normally distributed data. The difference here is that the temporal ordering is considered.



**Figure 1.** Diagram of four linear temporal process models at a lag of 1.

Note. “ $y_{\#}$ ” is an arbitrary observed variable at time ( $t$ ) or a lag of one ( $t - 1$ ). Subscripts “H” and “W” indicate two different individuals in a given dyad. “ $\eta$ ”s are latent variables. Please note all models could be extended for more variables or more lags. Please note that candidate relations are shown here for explanatory purposes and may not represent final estimated models.

The first examinations into dyadic interactions from a correlation-based perspective used standardized scores to arrive at a measure of synchrony between husbands and wives (Allison & Liker, 1982; Levenson & Gottman, 1983). Termed *reciprocity scores*, they indicated the extent to which a husband’s value on one variable predicted his wife’s value at the next time point and vice versa. A more general approach considers cross-correlation directly (Shumway & Stoffer, 2006). Here, the relationships at a lag of 1 between two given individuals  $h$  and  $w$  in a dyad  $i$  can be expressed as follows:

$$r_{hw,i} = \frac{\sum (y_{h,t-1,i} - \bar{y}_{h,i})(y_{w,t,i} - \bar{y}_{w,i})}{T-1(s_{y_{w,h,i}})}, \quad (1)$$

where  $r_{hw,i}$  is the correlation between the husband’s scores at a previous time period ( $y_{h,t-1,i}$ ) with the corresponding wife’s scores ( $y_{w,t,i}$ ) at the next time period (i.e., “wife reciprocity”). Similarly,  $r_{wh,i}$  would be considered “husband reciprocity,” an estimate of the relation between the wife’s scores at a lag and the husband’s subsequent scores.  $s_{y_{w,h,i}}$  represents the pooled standard deviation for husbands and wives.

These methods have predominantly been used in studies examining how psychophysiological linkage in dyads relates to qualities of marital relationships. For instance, researchers using the reciprocity scores (Levenson & Gottman, 1983) and cross-correlation (Gates, Gatzke-Kopp, Sandsten, & Blandon, 2015) approaches have found that marital dyads with high correspondence in their psychophysiological

measures across time (e.g., respiratory sinus arrhythmia, heart rate) tend to have greater levels of conflict and marital dissatisfaction.

**Recommendations and limitations.** While traditionally conducted on psychophysiological measures, correlation-based approaches can be used on any data for which there are an appropriate number of paired observations for each dyad. For the cross-correlation approach and numerous methods to follow, a researcher can generate lagged variables from the original variable set by deleting the first row of observations and inserting those columns to the original set. This can be done automatically with some software programs, such as SPSS. Then correlation coefficients can be estimated. A major drawback of correlational approaches is that they do not control for the influence of other variables or prior time points.

**Vector Autoregression.** Vector autoregression (VAR) models estimate directed (i.e., not correlational), lagged relations among the dyad-level variables (depicted in Figure 1B). For instance, in daily diary data the lag may be on the order of days, with predictions from the day before being a lag of 1. In the context of dyads, VARs indicate the degree to which individuals’ set of variables predict both their own and their partner’s values at a later time. Whereas correlation-based approaches discussed previously do not indicate directionality or take into account the influence of other variables, VARs enable researchers to examine the relations among variables after controlling for other variables. The VAR models are expressed as follows:

$$y_{h,t,i} = \upsilon_h + \sum_{l=1}^{l=L} \phi_{h,t-l,i} y_{h,t-l,i} + \sum_{l=1}^{l=L} \phi_{wh,t-l,i} y_{w,t-l,i} + \varepsilon_{h,t,i}, \quad (2)$$

$$y_{w,t,i} = \upsilon_w + \sum_{l=1}^{l=L} \phi_{w,t-l,i} y_{w,t-l,i} + \sum_{l=1}^{l=L} \phi_{hw,t-l,i} y_{h,t-l,i} + \varepsilon_{w,t,i}, \quad (3)$$

where, as before,  $y_t$  represents a vector of observed variables with  $h$  and  $w$  indicating the variables for husband and wife, respectively, in a given dyad  $i$ . The subscripts  $wh$  and  $hw$  indicate reciprocity much like in the description of cross-correlation, with  $wh$  representing the wife's observed value influencing the husband's at later time points, and vice versa for  $hw$ . The matrix  $\phi$  indicates the beta weights (much like in a regular regression equation) for the degree to which the variables at a given time point ( $y_t$ ) are predicted by the variables up to a certain lag  $L$  ( $y_{t-L}$ ). Large  $\phi$  values for the relation between the one individual at  $t-1$  with the other individual at  $t$  indicate that the first individual provides some predictive value for the second one (after controlling for the lagged influence the individual has on themselves). Values near zero would indicate a lack of association. The intercepts are denoted with  $\upsilon$ . The researcher can either decide a priori how far back in time to include, or  $L$  can be selected empirically by including all lags until additional lags fail to improve the model. The errors  $\varepsilon$  are assumed to have a zero mean, be normally distributed, and uncorrelated over time (Shumway & Stoffer, 2006).

VARs are the foundational approach for methods to follow, and while often used in other fields to examine how one party may influence another, they have not been applied to dyad-level data in psychology as frequently. However, one study used piecewise VAR models (equivalently called difference equations) to examine marital interactions during couples' discussion of conflictual issues. By examining the patterns of VAR results, they identified five types of marriages and were able to use these to significantly predict divorce (Gottman, Murray, Swanson, Tyson, & Swanson, 2005).

**Recommendations and limitations.** Most software programs used by social scientists have VAR capabilities. To name a few, SAS, SPSS, R, and Matlab all have time series options, and models can be estimated with SEM (structural equation modeling) software (Hamaker, Dolan, & Molenaar, 2002). VARs are readily implementable on data that are in the form of continuous scales. Measurements of behaviors, symptoms, or moods gathered in daily diary data may be optimal for this approach, as would most forms of psychophysiological data. Observational behavioral data can be used if the data are coded via a scale. A limitation of VARs is that they do not contain a measurement model. As such, any error in measurement is not taken into account.

**State-Space Modeling.** State-space modeling (SSM) incorporates a measurement model to quantify latent constructs that then can be modeled from within a VAR framework. In this way, these models can more precisely model the constructs of interest in the case where measurement errors may be present or when the constructs are best considered a weighted combination of indicators (i.e., a latent variable). This is particularly helpful when researchers have numerous items on a questionnaire that relate to the same underlying construct. Like VARs, SSMs can be applied to psychophysiological, observational, or daily diary scales. The measurement equation relating the observed  $y$  matrix of  $p$  variables to the latent  $\eta$  variables is as follows:

$$y_{h,t,i} = \tau_h + \lambda_h \eta_{h,t,i} + \zeta_{h,t,i} y_{w,t,i} \quad (4)$$

$$y_{w,t,i} = \tau_w + \lambda_w \eta_{w,t,i} + \zeta_{w,t,i}. \quad (5)$$

As written here, the  $\lambda$  loadings that relate the actors' observed variables to the latent construct can differ across dyads, similar to the idiographic filter introduced in dynamic factor analysis literature (Nesselroade, Gerstorf, Hardy, & Ram, 2007). In practice they are often set to be equivalent for the individuals. The latent variables can then be used in the dynamic (also referred to as transition or structural) equations:

$$\eta_{h,t,i} = \upsilon_h + \phi_h \eta_{h,t-1,i} + \phi_{hw} \eta_{w,t-1,i} + \varepsilon_{h,t,i}. \quad (6)$$

$$\eta_{w,t,i} = \upsilon_w + \phi_w \eta_{w,t-1,i} + \phi_{wh} \eta_{h,t-1,i} + \varepsilon_{w,t,i}. \quad (7)$$

The matrices are written separately here for wives and husbands for explanatory purposes and to directly map onto the depiction in Figure 1C. As with the VAR,  $\upsilon$  indicates the level, and  $\phi$  the matrices of lagged relation weights. Notation follows convention described for VAR above. Please note that by using the first-differenced variables (i.e., subtract previous time point from the present) the SSM model aligns with McArdle's (2009) latent change score model, where change can predict change, but as of yet this has not been used in dyadic time series data.

SSM can be conducted at the dyad level as demonstrated by Song and Ferrer (2009) on a romantically dating dyad for whom 60 days of affect scores were collected. In Chow, Mattson, and Messinger (2014), SSM was used to investigate mother-infant interactions on a sample where about half of the infants had an older sibling diagnosed with autism spectrum disorder. Using a group-based model they identified a lag of two best fit the data. From this model, they found that the infants' affect did not have a large influence on mothers' affect and vice-versa.

**Recommendations and Limitations.** SSM models can be fit using the *SsfPack* R package (Koopman, Shephard, & Doornik, 1999). State–space models offer the potential for much flexibility from a modeling framework beyond what was presented here. For instance, SSM models can accommodate time-varying relations by having parameters that change across time (Chow, Zu, Shifren, & Zhang, 2011) or identify categorical shifts in relations among variables (as in regime-switching models; Hamaker & Grasman, 2012). SSMs are also ideal when the researcher would like to include multiple measurements of the same construct, such as numerous items on a scale or behavioral reports from different sources.

**Unified SEM.** The unified SEM (uSEM, also referred to as structural VAR; Chen et al., 2011) incorporates contemporaneous effects into state–space and VAR models (see Figure 1D). As the name may suggest, uSEMs estimate effects from within the SEM framework (Kim, Zhu, Chang, Bentler, & Ernst, 2007). The ideal data for the uSEM approach is the same as that for VAR and state–space models. uSEMs can model either observed variables or latent (i.e., unobserved) variables from a dynamic factor analytic perspective (Molenaar, 1985). The measurement equation can be written identically as that for SSM (Equations 4 and 5).

The addition of contemporaneous effects can clearly be seen by adding the variables at time:

$$\eta_{h,t,i} = \nu_h + (A_{h,t,i}\eta_{h,t,i} + A_{hw,t,i}\eta_{w,t,i}) + \left( \sum_{l=1}^{l=L} \phi_{h,t-l,i}\eta_{h,t-l,i} + \sum_{l=1}^{l=L} \phi_{hw,t-l,i}\eta_{w,t-l,i} \right) + \varepsilon_{h,t,i}, \quad (8)$$

$$\eta_{w,t,i} = \nu_w + (A_{w,t,i}\eta_{w,t,i} + A_{wh,t,i}\eta_{h,t,i}) + \left( \sum_{l=1}^{l=L} \phi_{w,t-l,i}\eta_{w,t-l,i} + \sum_{l=1}^{l=L} \phi_{wh,t-l,i}\eta_{h,t-l,i} \right) + \varepsilon_{w,t,i}, \quad (9)$$

where the parameters described in VAR and dynamic state–space equations above are the same. The important addition that differentiates uSEMs is the **A** matrix, which contains contemporaneous effects. These effects indicate the extent to which values of another variable instantaneously relate to a given variable after controlling for lagged effects (but variables are not allowed to predict themselves contemporaneously). As with the VAR and SSM, the researcher can either decide a priori how many lags to include, or  $L$  can be selected empirically.

One study has used uSEM to better understand gendered social behaviors (Beltz, Beekman, Molenaar, & Buss, 2013). Coding behaviors in 10-second intervals, the researchers examined sex differences in sociotemporal

dynamics of play behavior in same-sex dyads. The results supported hypothetical differences in play between the genders: Girls' levels of positive affect were dependent on their peers' prior positive affect more so than boys', whereas for boys, the current level of vigor in activity was predicted by their peers' current levels of activity.

**Recommendations and limitations.** Unified SEMs can be estimated via either the *vars* (Pfaff, 2008) or *gimme* (Lane, Gates, & Molenaar, 2015) R packages and can also be conducted on observed variables (see Gates, Molenaar, Hillary, & Slobounov 2011). uSEMs are particularly useful when the researcher suspects that contemporaneous relations exist in the data or that the sampling rate may be too large to identify effects using solely the lags. Limitations for uSEMs are similar to some of those seen in correlational and VAR analysis. Additionally, estimation via SSM yields more precise values for the measurement parameters (Chow, Ho, Hamaker, & Dolan, 2010).

**Hidden Markov Models.** Hidden Markov models (HMMs) are similar to some of the previously discussed models in that they can have latent states and they model the temporal relations among these states. However, unlike traditional SSM, the latent states in HMMs and the relations among them are not continuous. HMMs estimate the discrete probability of going from one state (often defined as the presence of a set of behaviors) to another. Hence, HMMs are immediately well suited for categorical data.

Patterns of transitions between states in HMM use only information from the previous state. Thus HMMs have a lag of 1. There are  $N$  distinct states that in the context of dyadic interaction represent the state of the dyad as a whole. Hence, unlike the prior models where one individual's behavior can be modeled as predicting another individual's, HMMs describe how the two individuals in the dyad are relating at a given point in time as well as the probability of staying in that state or going to a different state. The measurement component is as follows:

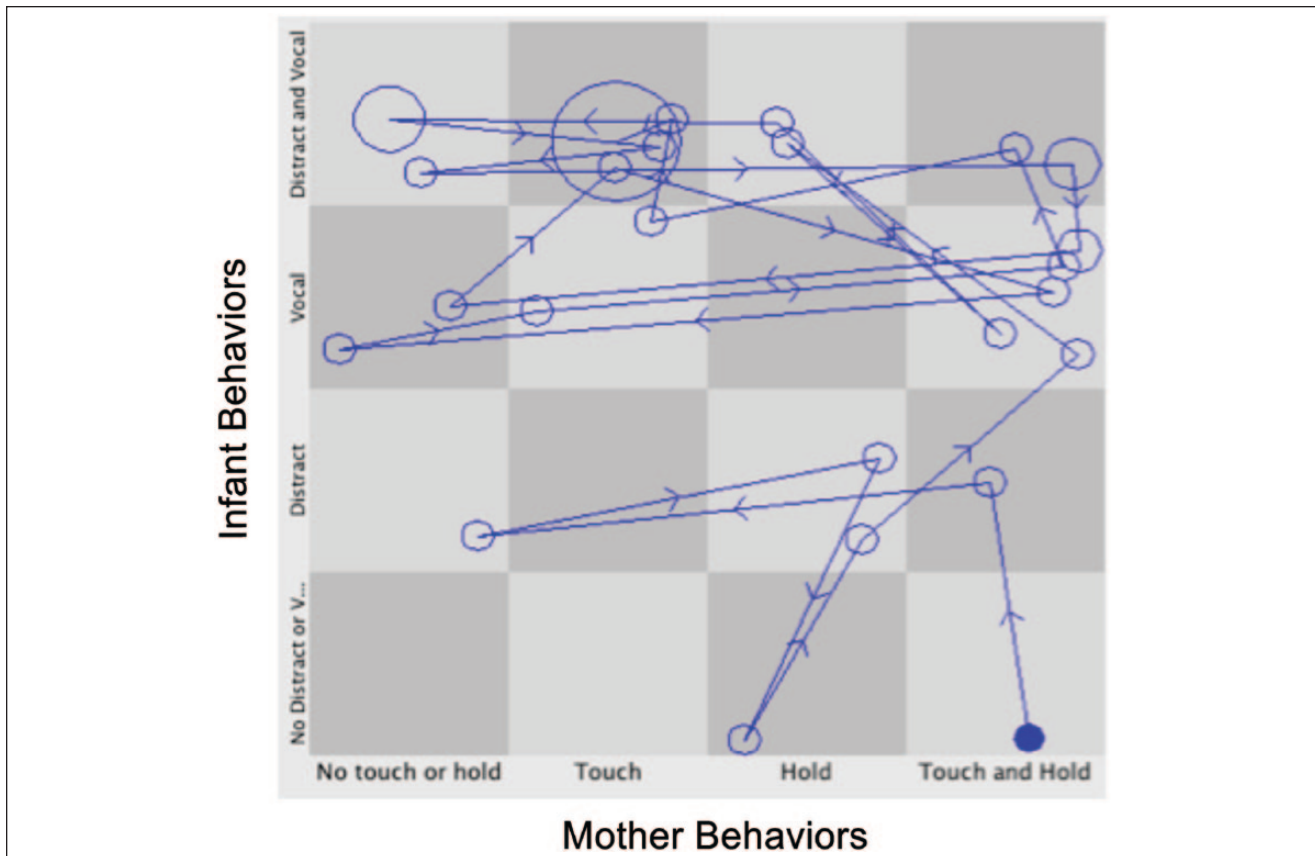
$$b_j(k) = Pr[v_k | S_j], \quad (10)$$

which indicates the probability of a given categorical observation  $v_k$  (e.g., smiling) occurring while in state  $S_j$ . The transition probability  $a_{ij}$  of transitioning from state  $S_i$  to state  $S_j$  is given by the following:

$$a_{ij} = Pr[q_t = S_j | q_{t-1} = S_i], \quad (11)$$

which can be considered a dynamic equation where  $q_t$  is the state the dyad is in at time  $t$  and  $q_{t-1}$  is the previous state the dyad experienced (adapted from Rabiner & Juang, 1986). Note that the probability of staying in the same state tends to be high in these models.

Variants of HMMs have been used in a number of contexts to examine mother–infant interactions. Rovine,



**Figure 2.** Two-dimensional depiction of mother ( $x$ -axis) and infant ( $y$ -axis) behaviors across time obtained using GridWare software. Note. Lines and arrows indicate transition trajectory from one state to another; circle size indicates length of time in one state for each visit; darkened circle represents the starting state.

Sinclair, and Stifter (2010) introduced the utility of using HMM for dyad-specific inquiries. A later study used the observed states (i.e., Markov modeling) and found that the transition from a fussing state in the infants was more likely to be followed by feeding in mothers who were heavier, and an intervention decreased this likelihood (Anzman-Frasca et al., 2013). Another study found that mothers' soothing behaviors changed from 2 to 6 months, with mothers using more sophisticated auditory soothing methods (Stifter & Rovine, 2015).

*Recommendations and limitations.* The R package *dep-mix* is a readily available and well-maintained resource for conducting HMM analysis (Visser, 2014). HMMs are ideal when the underlying research question relates to the “state” the dyad is in. This approach will identify how behaviors in one of the partners cohere with behaviors in another partner to arrive at one of the states that the dyad as a whole is experiencing simultaneously. One potential drawback is that it can be difficult to decide how many states to include.

*State-Space Grids.* Research questions sometimes require descriptive measurements (as opposed to models) that

capture dyads' categorical behaviors. As with the above state-space approaches, one can think of the dyads' behaviors as generating a state that they are mutually in. A freely distributed program for state-space grids (SSGs), GridWare, provides measures describing interaction patterns and a two-dimensional visualization (Lamey, Hollenstein, Lewis, & Granic, 2004). Figure 2 depicts the visualization grid where the  $x$ -axis contains behaviors of a mother with the  $y$ -axis containing behaviors of her infant. The circle sizes indicate the amount of time spent in a state with the arrows indicating to which state they transitioned next. One can immediately see, for instance, that the largest circle occurs where the mother is touching the infant (but not holding) and the infant is distracted and vocal.

GridWare arrives at descriptive measures such as how long the dyads are in a given state, which state they frequent the most, and how often they transition in and out of states. These measures can provide insights above and beyond traditional or aggregated measures (Moore et al., 2013). Most studies of dyadic interaction that use SSGs have focused on flexibility, which can be quantified in the three ways using GridWare: stability (average time in each state), fluctuation (number of transitions), and dispersion (total number of

states experienced). These measures have been used to demonstrate that rigidity of emotional states for parent–child dyads may be maladaptive in childhood and adolescence (e.g., Lunkenheimer, Olson, Hollenstein, Sameroff, & Winter, 2011; Van der Giessen, Branje, Frijns, & Meeus, 2013), yet flexibility during mother–infant interactions may not relate to secure attachment as hypothesized (Guo, Leu, Barnard, Thompson, & Spieker, 2015).

*Recommendations and limitations.* SSGs are particularly ideal for observationally coded behaviors that indicate the presence or absence of a given behavior during a dyadic interaction. Continuous variables cannot be included. While less mathematically complicated than some other approaches, this approach requires that much thought goes into generation of the data matrix.

*Recurrence Quantification Analysis.* Much like HMMs and SSGs, recurrence quantification analysis (RQA) has been predominantly applied in the psychological sciences on time series containing nominal data (e.g., observational coding). With RQA, researchers can identify the rigidity of the dyad as well as the temporal ordering of behaviors in a number of ways (Zbilut & Webber, 1992). At its simplest, RQA arrives at a “recurrence rate,” or the overall probability that states will reoccur. RQA also quantifies the length of time between states as well as the time spent in each state (much like stability above). Cross-RQA, much like cross-correlation discussed above, quantifies the relation between one actor’s behavior at a given time point with the other actor’s at an earlier time point. Another interesting measure, “trapping time,” indicates the average time spent in each state (Marwan, Kurths, & Saparin, 2007). Finally, RQA provides a measure of entropy for the time series as a to measure stability. Low entropy indicates highly predictable (stable) patterns with high entropy indicating behavioral patterns that require more information to predict.

These measures have been used to study a wide range of dyadic interaction. For example, entropy has been used to identify points of change in a longitudinal study examining the relation between parent–child interactions and treatment efficacy in aggressive children (Lichtwarck-Aschoff, Hasselman, Cox, Pepler, & Granic, 2012). Cross-RQA has been used to examine language development in children (e.g., Dale & Spivey, 2006), coordination of eye movements in viewing tasks (Richardson & Dale, 2005), and concordance of posture during conversation (Shockley, Santana, & Fowler, 2003).

*Recommendations and limitations.* RQA can be implemented using the Matlab toolbox “RQA” or the R package “CRQA” (Coco & Dale, 2014). RQA has been used predominantly on categorical or nominal behavioral codes. One advantage that RQA has over other techniques is that

it can be useful even for short or nonstationary time series. Some of the measures, such as entropy, may not be familiar to psychology researchers, and like SSG, the burden for the researcher is to be particularly mindful when preparing the data to ensure meaningful states.

*Differential Equations.* Differential equations models have been frequently used to study dynamic dyadic processes. These models are similar to VAR models in many ways, but they aim to predict changes of a system over time. Hence, the derivatives, or rate of change, are often specified as the dependent variables. For example, the following bivariate differential equations model can be used to represent dyadic interactions:

$$\frac{dy_h}{dt} = a_1 + b_1 y_h + c_1 y_w. \quad (12)$$

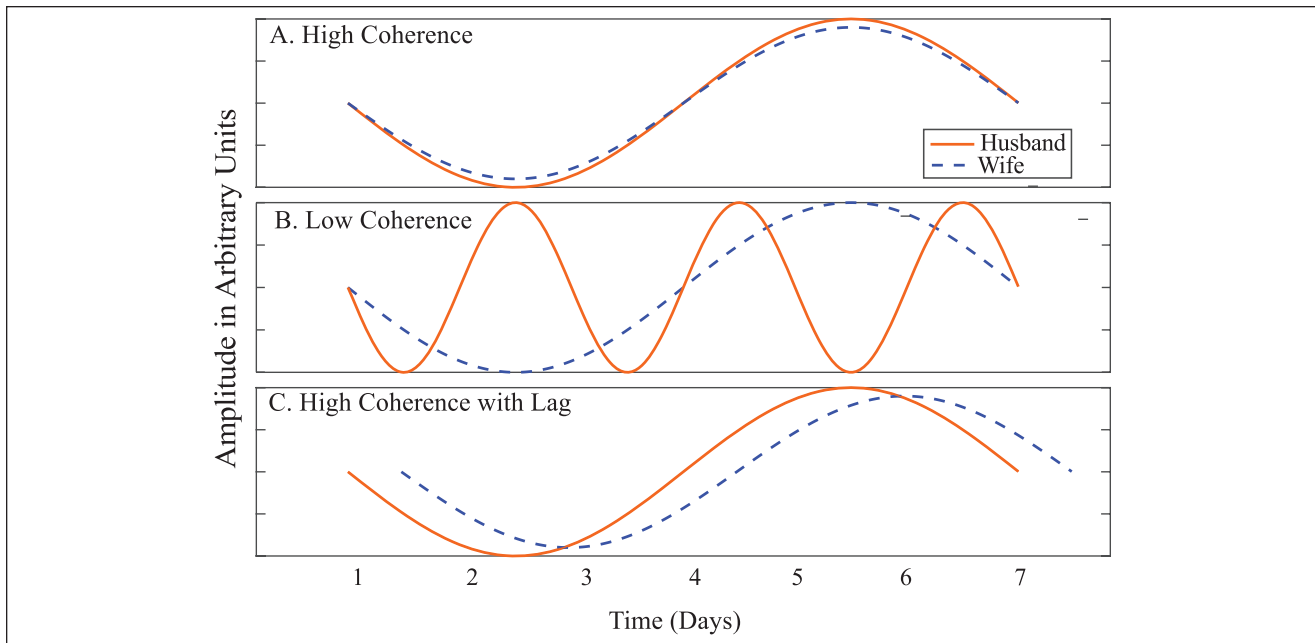
$$\frac{dy_w}{dt} = a_2 + b_2 y_w + c_2 y_h. \quad (13)$$

Here,  $\frac{dy_h}{dt}$  and  $\frac{dy_w}{dt}$  are rates of change in  $y_h$  and  $y_w$ , respectively. The parameters  $a_1$  and  $a_2$  represent the two variables’ equilibrium. The parameters  $b_1$  and  $b_2$  represent the extent to which  $y_h$  and  $y_w$  change depending on themselves, and  $c_1$  and  $c_2$  indicate the extent to which  $y_h$  and  $y_w$  change depending on each other. More complicated dynamic processes can be represented by alternative differential equations. For example, changes of a partner may be dependent not on the level of the partner but on the congruence between the two by using the difference score between  $y_h$  and  $y_w$  as a predictor (Felmlee, 2006; Felmlee & Greenberg, 1999).

Ferrer and Steele (2012) fitted four theoretically driven differential equation models to daily data from heterosexual couples and found that couples meaningfully differed in the best model describing their affective interactions. Other examples are found in Boker and Laurenceau (2006), Butner, Amazeen, and Mulvey, (2005), Ferrer and Helm (2013), and Ram, Shiyko, Lunkenheimer, Doerksen, and Conroy (2014).

*Recommendations and limitations.* Differential equations can be helpful when the researcher wishes to identify when one actor influences change in the other actor at a later time. SAS PROC MODEL is a flexible procedure for fitting these models, but selection of an appropriate functional form can be challenging if no strong theory exists, and estimation can be difficult.

*Frequency Domain Analysis.* Frequency domain analysis, also known as *spectral analysis*, includes a family of methods for examining cyclical patterns in data. In frequency analysis, time series data are decomposed into sinusoidal waves



**Figure 3.** Depiction of types of relations that can be quantified using coherence and directed coherence analysis.

with different frequencies. For each sinusoidal wave, its *power* reflects the amount of variance it accounts for in the data. Many physiological signals contain frequency bands that are associated with well-established clinical interpretations. For example, respiratory sinus arrhythmia (RSA), a measure of parasympathetic influence on heart rate, is typically obtained by computing the power of heart rate in the respiratory frequency band (0.12–0.40 Hz in adults). In some cases the power distribution of a signal is not known, and researchers may use frequency analysis to explore the dominant frequency bands in the process of interest. For instance, if a time series of daily diary data has a peak in the power estimate corresponding to 7 days, then a weekly effect has been discovered.

In the frequency domain a number of statistics can be obtained to study dyadic processes. For instance, the *coherence* is a frequency domain measure equivalent to a correlation. A high coherence at a particular frequency band indicates that two signals are both characterized by that frequency (Figure 3A). It can be used to examine whether two people show the same rhythmic patterns in social interactions, their psychophysiological measures, or any other data being used. Individuals with different dominant frequencies or cycles (Figure 3B) would have low coherence. If the individuals are indeed characterized by high coherence, then one can examine lead–lag relations in the dominant frequencies using these methods. This *phase* difference as depicted in Figure 3C indicates that the husband leads the wife in the frequency of interest.

By first fitting a VAR model to time series data, and then transforming the VAR model into the frequency domain,

one could obtain more reliable estimates of dynamic interactions (Schlögl, 2007). This transformation yields a number of statistics assessing causal lagged relations, such as the partial directed coherence (PDC; Baccalá & Sameshima, 2001) and its extended version, generalized partial directed coherence (gPDC; Baccalá & de Medicina, 2007). Like the cross-lagged coefficients in VAR, these estimates assess the degree to which one partner is influenced by the other partner's values at previous time points, controlling for autocorrelations. Unlike VAR, these effects are represented in terms of periodicities.

Gottman and Ringland (1981) are one of the first to use frequency analysis to examine dyadic interactions. They examined coherence and phase difference in changes of attentional and affective involvement in three mother–infant dyads through videotaped episodes of face-to-face interactions. Following this study, two groups used coherence and phase to examine synchrony in mother–infant interactions. Applying these descriptive measures to group-level analysis, they found significant differences between term and preterm babies (Lester, Hoffman, & Brazelton, 1985), and between depressed and nondepressed mothers (Field, Healy, Goldstein, & Guthertz, 1990). In a recent article, Liu and Molenaar (2016) use PDC and gPDC to examine interaction patterns in skin conductance responses between an autistic child and his therapist during psychotherapy.

**Recommendations and limitations.** Descriptive statistics in the frequency domain, such as coherence and phase, can be obtained in standard statistical packages including SAS,



Matlab, and R. Model-based statistics, such as PDC and gPDC, can be conducted using publicly available Matlab tool boxes (e.g., eMVAR or BioSig). One drawback of frequency domain statistics is that their interpretations are not as straightforward as the time domain models when used with data without obvious cyclical patterns.

### Group-Level Methods

Clinicians may wish to conduct group-level analysis instead of (or in addition to) conducting dyad-level analysis. Models that describe the group may be used to arrive at inferences that typify their clientele base or can generalize to the population from which they participants are drawn. In this way, group-level models may aid in understanding what is a “typical” temporal process for a given situation. All the models described above for dyad-level analysis can be conducted to arrive at one group model by vertically concatenating dyads’ multivariate time series. However, for reliable and useful models to be obtained concatenating requires that the dyads all have identical processes, an assumption that often is not met (see Molenaar, 2004). The approaches below allow for group-level models that do not assume homogeneity.

**Multilevel Models.** Multilevel models are also called hierarchical linear models (HLMs; Raudenbush & Bryk, 2002) and mixed effects models (Laird & Ware, 1982). They provide two types of estimates: *fixed effects*, which represent the average trends in the sample, having identical values across dyads; and *random effects*, which can vary across dyads and are assumed to be normally distributed in the sample. With the two effect types, multilevel models allow for generalizable inferences as well as insight into differences across dyads.

The number of levels in multilevel models depends on the nested structure in the data. With dyadic data it is common to use a two-level model, where repeated measures are nested within the dyad and a between-dyad level captures differences across dyads. The multilevel modeling framework is highly flexible and allows researchers to quantify interpersonal interactions in various ways. For example, the multilevel growth curve model can be used to represent the trajectories of the outcome variables as functions of time, such as polynomial and even nonlinear functions (e.g., exponential). Depending on the function form, the covariance between random components captures similarity in different aspects of the overall shape of their data. In some cases, however, researchers may be more interested in interpersonal dynamics at a finer time scale, rather than the overall shape. For example, if it is hypothesized that husband and wife have similar values from one time point to the next, the following model can be used:

$$\begin{array}{ll} \text{Level 1 (within-dyad level):} & \text{Level 2 (between-family level)} \\ H_{it} = \pi_{0i} + \pi_{1i}WW_{it} + \varepsilon_{it} & \pi_{0i} = \beta_{00} + \beta_{01}WM_i + u_{0i} \\ & \pi_{1i} = \beta_{10} + u_{1i} \end{array} \quad (14)$$

In this model,  $H_{it}$  represents the value from husband in dyad  $i$  at time  $t$ ,  $WW_{it}$  represents the deviation of the wife’s value from her person-mean at time  $t$ , and  $WM_i$  represents her person-mean (i.e.,  $W_{it} = WM_i + WW_{it}$ ) over time. Hence,  $\pi_{0i}$  is the estimated person-mean for the husband in dyad  $i$ , and  $\pi_{1i}$  represents the degree of synchrony for dyad  $i$ . A positive  $\pi_{1i}$  indicates that if the husband’s value at a given time point is higher than his average, the wife’s value at the same time point is also higher than her average. Notably,  $\pi_{1i}$  can vary across dyads, enabling the testing of possible covariates. For example, if a relationship satisfaction measure is hypothesized to influence the degree of synchrony, it can be added as a Level-2 predictor of  $\pi_{1i}$ .

This modeling framework can be easily extended to test for other mechanisms of interpersonal interactions. For instance, if it is hypothesized that one partner’s value at a particular time point predicts the other partner’s value at the following time point, a multilevel VAR model can be used. If the underlying dynamics of interaction can be represented by a set of differential equations, multilevel differential equations models can be tested.

Reed, Randall, Post, and Butler (2013) used multilevel models to examine positive and negative physiological linkage between romantic partners and the moderation effects of partner influences. They measured blood pressure, interbeat interval, and skin conductance from 44 heterosexual couples, finding that couples who reported low partner influence in health behaviors showed negative linkage in their blood pressure. Similarly, Liu, Rovine, Klein, and Almeida (2013) used multilevel models to examine synchrony in diurnal cortisol patterns, a physiological indicator of stress, between spouses with daily diary data. They found that spouses synchronize their diurnal cortisol slope, such that on days when one experiences a faster decline in diurnal cortisol than usual, the spouse also experiences a faster decline. In addition, positive synchrony in cortisol awaking response was observed only in couples reporting high levels of marital strain and disagreement.

**Recommendations and limitations.** Multilevel modeling can be carried out in a variety of statistical programs. Sample code for SAS, SPSS, and Mplus can be found in Bolger and Laurenceau (2013). Although multilevel models can accommodate heterogeneity among dyads by estimating random effects, they are constrained in the sense that all subjects have to follow the same fixed-effects model. This may not be ideal if the process differs across dyads.

**Group Iterative Multiple Model Estimation.** While multilevel models arrive at dyad-level estimates for a constrained set

**Table 2.** Methods for Group-Level Modeling.

Method	Research questions answered	Recommendations and limitations	Implementation
Multi-level	What is the average magnitude for which dyads covary? How do individual dyads differ from this?	All dyads must conform to the same model; excellent for continuous data	Standard statistics software
GIMME	What are the best aggregate- and individual-level models for how dyads covary? What are the dyad-level estimates?	Individual-level models may be driven by noise; excellent for continuous data	R package: gimme (Lane, Gates, & Molenaar, 2015) GIMME Matlab toolbox (Gates & Molenaar, 2014)
Dynamic corr.	Do dyads tend to covary across time according to a shared a nonlinear trajectory?	Group-level statistic that does not give insight into individual dyads; excellent for continuous data	R package: dynCorr (Dubin, Qiao, & Müller, 2012) R program (Liu, Zhou, Palumbo, & Wang, 2016)

Note. Multi-level = multilevel modeling; GIMME = group iterative multiple model estimation; Dynamic corr. = dynamic correlation.

of relations, group iterative multiple model estimation (GIMME) arrives at patterns of effects that exist for the group as well as patterns that are unique for the dyad (Gates & Molenaar, 2012). In this way, GIMME attends directly to the issue of heterogeneity in temporal processes across. GIMME can be used on continuous data such psychophysiological and daily diary data. Importantly, GIMME does not require that homogeneity among individuals exists in terms of (1) the structure describing their models or (2) the magnitude of the effects.

GIMME begins by arriving at a pattern of temporal effects that describes the sample. These are then used as a prior for dyad-level paths, an approach that has been shown to greatly improve the ability to recover the presence of temporal effects describing the relations among variables for the dyad-level time series (Gates & Molenaar, 2012). Having arrived at patterns of temporal effects that exist at the sample and dyad levels, GIMME estimates the effects for each  $i$  dyad. The uSEM model (Equations 7 and 8) can be extended:

$$y_{h,t,i} = (A_{h,i} + A_{h,i}^g)y_{h,t,i} + (A_{hw,i} + A_{hw,i}^g)y_{w,t,i} + (\phi_{h,i} + \phi_{h,i}^g)y_{h,t,i-1} + (\phi_{hw,i} + \phi_{hw,i}^g)y_{w,t-1,i} + \varepsilon_{h,t,i}, \quad (15)$$

$$y_{w,t,i} = (A_{w,i} + A_{w,i}^g)y_{w,t,i} + (A_{wh,i} + A_{wh,i}^g)y_{w,t,i} + (\phi_{w,i} + \phi_{w,i}^g)y_{w,t,i-1} + (\phi_{wh,i} + \phi_{wh,i}^g)y_{h,t-1,i} + \varepsilon_{w,t,i}, \quad (16)$$

where subscripts  $h$  and  $w$  indicates the husband and wife, respectively, for the  $i$  dyad, and superscript  $g$  denotes group-level patterns of effects. Matrix notation is the same as described for uSEM above. Note that all estimates are at the dyad level.

GIMME has predominantly been used to look at synchrony in brain regions (e.g., Yang, Gates, Molenaar, & Li, 2015). However, increasingly researchers are using this approach to look temporal patterns of individuals across time. For example, the social interaction study noted above

(Beltz et al., 2013) conducted analysis for each individual separately, and the same data could be used in GIMME to enable identification of group-level effects.

**Recommendations and limitations.** GIMME is freely available for download as an R package (Lane et al., 2015) and Matlab toolbox (Gates & Molenaar, 2014). As with cross-correlation, VAR, uSEM, and SSM, at this time GIMME is currently best for data that do not have cyclical relations such as weekend effects that may surface in substance use behaviors. Providing that they are not constructs of interest, cyclical relations can be regressed out prior to analysis.

**Dynamical Correlation.** Dynamical correlation is a correlation index for quantifying the similarity between two sets of curves (Dubin & Müller, 2005). It was developed in the framework of functional data analysis, an emerging field in statistics to analyze data in the form of a curve or function. If we conceptualize a psychological process of interest as a continuous smooth function of time, longitudinal data can be viewed as observations taken from this function at discrete time points, possibly contaminated by measurement errors. In many cases, researchers are interested in studying the underlying psychological process, not observations at discrete time points. The goal of functional data analysis is thus to obtain statistics describing relations among underlying functions, which can be recovered if measurements are dense.

Given two intensively measured variables from a sample of dyads, the covariation between their underlying functions can be quantified by dynamical correlation. To some degree, dynamical correlation is a proxy of Pearson correlation in functional data analysis. It is a standardized correlation measure within the range of  $[-1, 1]$ . In Pearson correlation, observations first need to be centered by subtracting the sample mean from each observation. Similarly, in dynamical correlation, the curves need to be centered by subtracting the sample mean curve from each curve. Then,

a correlation index is created to indicate the similarity between the centered curves. In this regard, dynamical correlation is also conceptually similar to the multilevel growth curve modeling approach, which quantifies synchrony with a covariance between random effects (i.e., deviations from the sample means). However, dynamical correlation is more flexible in that it does not require a functional form to be specified. Hence, it is convenient when modeling processes that do not show a regular, universal change trajectory. In addition, it does not place any assumption on the distribution of the centered curves, unlike multilevel models that require normality of random effects. As a result, dynamical correlation can be easily estimated and tested without any model convergence problem.

Liu, Zhou, Palumbo, and Wang (2016) illustrate the use of dynamical correlation in studying interpersonal physiological synchrony. Skin conductance was measured from heterosexual couples during two experimental conditions: (1) partners were seated back-to-back silently and (2) partners were seated face-to-face silently. Dynamical correlation between partners' skin conductance data was significant only in the face-to-face condition, which suggests that non-verbal conditions were sufficient for physiological synchrony to develop but only when visual cues were present.

**Recommendations and limitations.** The R package *dyn-Corr* can be used to conduct dynamic correlation analysis (Dubin, Qiao, & Müller, 2012) and an R program created for social scientists can also be obtained from Liu et al. (2016). Unlike multilevel models and GIMME, dynamical correlation is a group-level statistic and does not provide dyad-level estimates. Hence, it is not suitable for studying individual differences in dyadic interactions.

## Discussion

Studying how partners in dyads relate to each other across time from within a dynamic systems framework has already led to a wealth of insights that would not have been revealed in traditional cross-sectional research. Some guiding principles when selecting the appropriate modeling framework relate to the qualities of the data and the hypotheses. If one were interested in linear aspects of the data, for instance, how a partner's mood on one day influences a partner's mood on another day, then the class of models depicted in Figure 1 would be appropriate (i.e., cross-correlation, VAR, SSM, and unified SEM). For examination of specific periodic or cyclical properties of their data, such as weekly patterns or aspects of heart rate (see Figure 3), frequency domain analysis would be appropriate. Specific approaches, such as HMM, can be conducted when the researcher wishes to identify when a dyad shifts into a different interactive state, and GridWare or RQA can be used to quantify the flexibility and other measures of shared dyadic

interaction. Of the methods discussed here, HMMs, RQAs, GridWare, and SSM are immediately well-suited for non-stationary data, or when the relations among variables changes over time.

New terminology to describe dyadic relations has emerged concomitant with the increased use of these data types and emerging methods. For instance, psychophysiological literature has used the terms *synchrony* and *linkage* to describe shared covariation in dyads (Levenson & Gottman, 1983), whereas panel longitudinal models sometimes use the term *interdependence* to describe relations between individuals (Cook & Kenny, 2005). *Coregulation* (Butler & Randall, 2013) and *compensatory* (Gates et al., 2015) have been used to define dyadic relationships that contribute work together to return to emotional stability. Reed et al. (2013) offer a unified set of terms to describe different types of covariation, using "in-phase" to describe dyads that have a positive relations with each other and "anti-phase" to denote negative relations. As the use of these techniques increases, it is likely that more nuanced terms will become available. However, it is helpful to have general terms that are consistent in usage in combination with these methods to help with comparisons of results and hypotheses across papers. The present article provides an overview of currently available methods that will likely be the foundation for methods that are developed as researchers continue to use and push the boundaries of traditional time series techniques to best answer their research questions of interest.

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