Inferring functional connectivity in MRI using Bayesian network structure learning with a modified PC algorithm

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Abstract

Resting state functional connectivity MRI (rs-fcMRI) is a popular technique used to gauge the functional relatedness between regions in the brain for typical and special populations. Most of the work to date determines this relationship using Pearson’s correlation on BOLD fMRI timeseries. However, it has been recognized that there are at least two key limitations to this method. First, it is not possible to resolve the direct and indirect connections/influences. Second, the direction of information flow between the regions cannot be differentiated. In the current paper, we follow-up on recent work by Smith et al. (2011), and apply PC algorithm to both simulated data and empirical data to determine whether these two factors can be discerned with group average, as opposed to single subject, functional connectivity data. When applied on simulated individual subjects, the algorithm performs well determining indirect and direct connection but fails in determining directionality. However, when applied at group level, PC algorithm gives strong results for both indirect and direct connections and the direction of information flow. Applying the algorithm on empirical data, using a diffusion-weighted imaging (DWI) structural connectivity matrix as the baseline, the PC algorithm outperformed the direct correlations. We conclude that, under certain conditions, the PC algorithm leads to an improved estimate of brain network structure compared to the traditional connectivity analysis based on correlations.

Keywords: Resting state functional connectivity MRI (rs-fcMRI), Directed functional connectivity, Effective connectivity

Introduction

Resting-state functional connectivity MRI (rs-fcMRI), measures intrinsic, temporally correlated low-frequency BOLD signals in subjects at rest (not performing an explicit task) (Biswal et al., 1995). The measurement has shown increased popularity in recent years to examine the functional relatedness of independent brain regions and various forms of network phenomena due to its relatively straightforward implementation in clinical and non-clinical samples.

However, while rs-fcMRI has become a useful tool to examine brain organization there are at least two important considerations regarding typical approaches to operationalize functional connectivity that indicates need for improvement. These stem from the fact that, many investigations to date simply apply Pearson’s correlations to determine whether two regions are “functionally connected”. While this measurement of functional relatedness has offered powerful insights into brain organization (Fox and Raichle, 2007; Van Dijk et al., 2010), there are two clear limitations. One limitation relates to the ability to differentiate direct and indirect influences. The second limitation pertains to its inability to clarify the direction of information flow (i.e. directed functional connectivity (Stephan and Friston, 2010)).

As an illustration of indirect relationships between two nodes we use an example of Flu and Hay fever. Because flu and hay fever often co-occur during the same seasons of the year, they are correlated, leading a naïve observer to potentially conclude that hay-fever causes flu, or that flu causes hay-fever. In other words, the two separate entities are directly linked (or in our case functionally connected) as illustrated in Fig. 1. However, these constructs are not directly linked. Hay fever doesn’t cause the flu, nor does the flu cause Hay fever. It is only through a shared cause, in this case seasonal changes, that Hay fever and the Flu are correlated or connected statistically (Koller and Friedman, 2009).

It is likely, and indeed has already been demonstrated in both human (Habeck and Moeller, 2011; Honey et al., 2009; Zalesky et al., 2012), and animal work (Vincent et al., 2007) that these sorts of indirect influences account for some of the functional connectivity
measurements determined with traditional fcMRI methods. For example, Vincent et al. (2007) demonstrated how segments of the left and right primary visual cortex, known to lack direct anatomical connections (Van Essen et al., 1981), show strong functional connectivity. The authors show that this relationship is likely mediated through polysynaptic pathways in the brain.

Fig. 2 provides a schematic of the likely phenomenon.

With regard to directed functional connectivity, (i.e. determining who is directing who), traditional correlation methods cannot decipher the direction of the association. Many attempts to deduce this information via other methods have been noted, but the accuracy of these methodologies has recently been challenged (Friston, 2011; Smith et al., 2011).

In examining different approaches to determine directed functional connectivity, Smith et al. (2011) used simulated BOLD data to determine the quality of several common methods that attempt to identify A) true structure of a network (direct and indirect relationship), and B) causality within the network (who is directing who). The methods examined were correlation, partial correlation, inverse covariance (ICOV), Patel’s conditional dependence approach and selected Bayesian network (Bayes Net) methods including CCD (Cyclic Coordinate Descent), CPC (Conservative PC), FCI (Fast Casual Inference), PC and GES (Greedy Equivalence Search) (Smith et al., 2011). The partial correlation, ICOV and the Bayes Net methods performed well at identifying the structure of the system and Patel’s τ was the best at identifying the direction but there was no method that did well in identifying both the structure and the directionality.

Important, much of the work in Smith et al. (2011) focused on determining how well structure and causality could be detected across single subject simulated data. However, the noise inherent in a single subject’s data may lead to reduced performance of the measurements versus the more typical group-based analyses that many investigators apply. While there are certain drawbacks to examining group averaged matrices, where one first generates one composite matrix by averaging across the single subjects, it may assist performance on some of these measurements.

In this paper, we focus on re-examining one of the prior approaches highlighted in Smith et al. (2011), using two sets of simulated data but on group average matrices (average of all the subjects’ connection matrices). We also examine performance of the approach on empirical data. In this latter case, structural connectivity matrices based on diffusion-weighted imaging (DWI) served as our baseline. While the ability to ascertain causal relationships is still evolving, Bayesian inference can provide a reasonable estimate based on observed statistical evidence as indicated above. These methods are particularly powerful in certain networks where the ambiguities like feedback and forward connections are limited. This perspective is adopted here and we exploit a conditional independence test to infer probabilistic dependence between observed variables.

The Bayesian approach has been successful in applications like finding causal relation in protein signaling, in cellular networks, to drug design and more (Friedman, 2004; Gertrudes et al., 2012; Sachs et al., 2005) suggesting it warrants closer scrutiny for brain imaging data as well. We used the PC algorithm developed by Sprites et al. (2000) to determine the direct and indirect relationship and the directed functional connectivity in the examined systems.

Methods

We used two simulation data sets and one empirical data set as follows.

**Simulation 1**

Simulation 1 includes the data set (n = 50) designed by Smith et al. (2011). The neural network model in the simulation was based on the dynamic causal modeling (DCM) which was then fed through a non-linear balloon model for the vascular dynamics. The neural network model was defined as

\[ z' = \sigma Az + Cu \]

where z is the neural timeseries, z’ is its rate of change, u is the external inputs, C the weights controlling how the external inputs feed into the network, A has the network connections between nodes and \( \sigma \) is
the neural lag. This neural signal was fed into the balloon model for which the parameters are set in DCM.

The neural lag was set to 50 ms as a realistic estimate of the timelag in neural signaling. Twenty eight data sets were created with different number of nodes, different TR and different percent of noise. Table 1 shows the parameters for the data we used from the Smith et al. (2011) data set.

Here we chose network simulations of varying size (i.e., 5, 10, and 50 nodes) like the other groups (Ramsey et al., 2011; Smith et al., 2011), except that Gates and Molenaar (2012) did not use data with 50 nodes. We also chose a subset of simulations that varied in session duration as there is significant variability in the duration of resting state sessions across connectivity studies. We also decided to examine a network with cyclic connections. Cyclic connections present links that start and end on the same vertices, and as mentioned below are often present in biological systems. The set, which includes cyclic connections, may be particularly challenging for structure learning because Bayesian networks are not optimized to support cyclic relationships. Last we decided to include a set that has feedback or backward connections in the system, as most biological links examined with rs-fcMRI are likely to have both feedforward and feedback connections.

Simulation 2

Another data set was created that explicitly models contemporaneous relations separately from the effects which occur at a lag of one TR (Gates and Molenaar, 2012). Since fMRI data is known to contain sequential dependencies (i.e., cross-correlation and autoregressive effects), it is important to test the present method’s ability to accurately recover the true relations and not be biased by these lagged influences. The data was created using Monte Carlo simulation for the following model

$$\eta_t = A\eta_t + \sum_{q=1}^{Q} \Phi_q \eta_{t-q} + \zeta_t$$

where $\eta_t$ is the ROI time series, $A$ is the contemporaneous relation among ROI, $\Phi$ is the lagged relation, and $\zeta_t$ is the error/white noise with variance 1 (Finaieh and Ljung, 1987; Gates et al., 2011; Kim et al., 2007; Mathews and Sicuranza, 2000) (Table 2).

We used three data sets for a 10 node network. Each had the same structure depicted in the 10 node graphic in Fig. 4. One network featured contemporaneous relations between the nodes, second network featured lagged relations and third network featured contemporaneous relations among the nodes and the autoregressive relation for each node (i.e., how the signal at a given time point predicts the signal of the same node at the next time point).

Empirical data

To obtain empirical data we used data from 8 healthy subjects, seven females and one male, with average age of 26.5. Imaging was performed during a single session for each subject on a 3 T Siemens Tim Trio scanner with a 12-channel head coil. Data acquisition included a T1-weighted image for anatomical reference, a functional MRI scan, a T2-weighted image, and a high-angular resolution diffusion weighted image (HARDI).

### Table 2

<table>
<thead>
<tr>
<th>Data set</th>
<th>Nodes</th>
<th>Relation</th>
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<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>Contemporaneous</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>Lagged</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>Contemporaneous &amp; lagged</td>
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**Diffusion-weighted imaging**

A HARDI scan was performed using an EPI sequence consisting of 72 gradient directions with $b$-value = 3000 mm$^2$/s along with 10 unweighted B0 images. Acquisition parameters for the scan included the following: TR = 7100 ms, TE = 112 ms, 2.5 mm$^3$ voxels, 48 slices, FOV = 230 × 230 mm. Diffusion data processing was carried out by connectomemapper (http://www.connectomics.org/connectomemapper/), and consists of four stages: coregistration of the T1-weighted image and B0 image, diffusion data reconstruction, tractography and identification of connections.

**Coregistration of T1-weighted and B0 image.** To facilitate accurate registration of the T1-weighted anatomical image onto the B0 image of the diffusion-weighted data, a T2-weighted image was also acquired (TR = 3200 ms, TE = 497 ms; 1 mm$^3$ voxels, 160 slices, FOV = 256 × 256 mm) as an intermediary. We performed a rigid-body transformation of the T1-weighted image onto the T2-weighted image, and then nonlinear registration of the T2-weighted image onto the B0 image. The additional step of nonlinear registration allowed us to account for image distortion common in diffusion-weighted data, such as susceptibility artifact, B0 field inhomogeneity, and eddy-current distortions. Skull-stripping was also performed on the T2-weighted and B0 images prior to this step to ensure robustness of the nonlinear algorithm. Every subject was then manually inspected to ensure high-quality accuracy for each step in the registration procedure.

**Diffusion reconstruction.** Diffusion data was resampled into 2 mm$^3$ voxel size and reconstructed using a Q-BALL scheme (Tuch et al., 2003) into an orientation distribution function (ODF) at each voxel. The ODF was defined on a tesselated sphere of 181 vertices, and represents the estimated diffusion intensity in each direction. At each voxel, we defined up to 3 directions of maximum diffusion as defined by the local maxima of the ODF. This step is analogous to computing the principal eigenvector when using the Diffusion Tensor Imaging (DTI) model.

**Tractography.** At each voxel of white matter, we initiated 32 evenly-spaced fibers for every direction of maximum diffusion. Each fiber
Fig. 3. Modified PC algorithm: We identify the probabilistic dependencies that were consistently identified across different settings of the parameters of the algorithm. The parameters were modified as shown here.

Fig. 4. The structure and causality of the Smith et al. data (2011): three network structures were chosen from Smith et al. (2011) to test the capability of the algorithm. The first one has 5 nodes, second one has 10 nodes and the third network has 50 nodes.
was propagated in opposite directions, and upon reaching a new voxel, continued in the direction of whichever maximal diffusion direction was closest to its current direction. The growth process of a fiber was stopped whenever this resulted in a change of direction sharper than 60°, or when its ends left the white matter mask. Additionally, fibers shorter than 20 mm in length were considered potentially spurious and were removed. This resulted in a large sample of reconstructed white-matter fibers across the whole brain.

Connection identification. Structural connections between cortical ROIs were identified by combining the results of the tractography with the cortical parcellation. For example, two ROIs i and j were said to be structurally connected if there existed a fiber with endpoints in i and j. Connections were weighted by the total number of fibers between two ROIs [http://www.connectomics.org/connectomemapper/].

Resting-state functional connectivity (rs-fMRI)
Functional data was acquired using a gradient-echo echo-planar imaging (EPI) sequence with the following parameters: TR = 2500 ms, TE = 30 ms, FA = 90°, 3.8 mm³ voxels, 36 slices with interleaved acquisition, FOV = 240 × 240 mm. Subjects were instructed to remain still and passively fixate on a crosshair for 10–20 min.

Processing. The raw fMRI data underwent standard fMRI preprocessing including slice-time correction, debanding, motion–correction, registration onto the T1 image, and resampling into 3 mm³ voxel size. Several additional steps were also taken to prepare the data for connectivity analyses (Fox et al., 2005), including temporal bandpass filtering (0.009 Hz < f < 0.08 Hz), spatial smoothing (6 mm full-width at half-maximum), and regression of nuisance signals. The latter includes the whole-brain signal, signals from ventricular matter and white matter, and the six parameters related to rigid-body motion correction.

Motion censoring. Subjects underwent several rigorous steps to correct for head motion during scanning. First, frame-to-frame displacement (FD) was calculated for every time point. FD was calculated as a scalar quantity using a formula that sums the values for framewise displacement in the six rigid body parameters (FDi = |Δdx| + |Δdy| + |Δdz| + |Δdx| + |Δdy| + |Δdz|, where Δdx = d(i – 1)x – dix, and similarly for the other five rigid body parameters) (Power et al., 2012). At each time point, if the FD was greater than 0.2 mm, the frame was excluded from the subject’s time series, along with one preceding frame and the two following frames (Power et al., 2012).

Connectivity. Time series were then computed for each of the cortical ROIs by averaging the signal intensity across all voxels within the ROI for each time point. Cross-correlations were computed between the time series of all ROI pairs.

Modularity analysis
To divide our data set (see T1-weighted structural MRI section) into specific brain networks we applied a community detection algorithm, as introduced by Newman (2006), to the structural matrix generated from our tractography procedures. This method uses modularity, a quantitative measure of the observed versus expected intra-community connections, as a means to guide assignments of nodes into specific communities or networks. The method by Newman was selected based on prior evidence that it is efficient, accurate, and commonly applied to both structural and functional brain data (Fair et al., 2009; Power et al., 2011; van den Heuvel et al., 2009; Van Dijk et al., 2010). The community detection algorithm splits our large scale system of structural data (DWI data) into 5 networks: A presumptive default network, fronto-parietal network, cingulo-opercular network, visual network and motor network. We examined the first 3 of these networks here due to the distributed nature of their connectivity and the nodes that defined them (see Fig. 8). The functional correlation and PC algorithm results were then compared to the network formed by the modularity analysis. That is, we clarified whether the functionally connected nodes (by both Pearson’s correlation and PC algorithm) matched with the structurally connected nodes within a given network using the structural DWI result as the baseline.

Conditional independence and PC algorithm
PC algorithm is an efficient method to learn a probabilistic graphical model that can generate the observed data. A probabilistic graphical model G consists of nodes (xi) representing the random variables and edges representing the probabilistic dependencies between the variables. Fig. 1 shows an example of Bayesian network where season, hay fever and flu are three variables/nodes and the connection between season and the two illnesses represents the edges/cause relationship.

A joint probability distribution is the combined probability of all possible events to occur. A very useful property of Bayesian networks is that the joint probability of observing all the variables decomposes nicely into factors, with one factor per variable. Each factor is a conditional distribution of a variable given its parents, \( P(x_i|\text{pa}_i) \), where \( \text{pa}_i \) represents the parents of \( x_i \). Thus, the joint probability is given as below

\[
P(x_1, \ldots, x_n) = \prod_i P(x_i|\text{pa}_i).
\]

Two events X and Y are said to be conditionally independent given a third event Z whenever the knowledge of Z makes X and Y statistically independent. For the network shown in Fig. 1, the joint probability distribution for hay fever would \( P(F|S) \) and \( P(H|S) \) (Pearl, 1995; Sachs et al., 2005). PC algorithm (Sprites et al., 2000) incorporates this notion of conditional independence. The algorithm begins with Step 1: a complete undirected graph Step 2: Edges between the nodes are deleted based on the conditional independence test. As illustrated in the example given node S, Node F and Node H are conditionally independent and hence the edge between F and H is deleted. Node S is saved as a separation set (Sepset) \( S[F,H] \) and \( S[H,F] \). In Step3, the v-structures are oriented
first and in Step 4: the remaining edges are orientated using the rules mentioned in sprites et al. (2000).

The function pc() in R (Kalisch et al., 2012) implements the four steps mentioned above.

Algorithm

For simulation 1, the correlations between nodes were created for every subject individually. Using the Schmidt–Hunter method for meta analyses, the correlation coefficient \( r \) between the nodes were averaged across all the subjects which was used as an input to the PC algorithm in R (Kalisch et al., 2012).

The procedure generates a matrix called pMax with a dimension of \( N \times N \) \( (N \) equals the number of nodes/vertices) for a given alpha value. Alpha is the tuning parameter for the PC algorithm. pMax is the maximum p-value of all the conditional independence tests between two given nodes. At any given alpha level, if the pMax for any two given nodes is less than the alpha value, there exists a connection between those two nodes. Based on this, the pMax matrix was converted into binary form. Importantly, the PC algorithm is threshold (alpha) dependent (a similar problem as with traditional correlation analyses). As such, using optimum threshold is very important as the results can be biased. So we focused here on developing a method that would not be based on a single threshold. We start this procedure with the binary matrix being created for varying alpha levels starting from 0.0 to 1 with an increment of 0.01. The intuition behind our approach is based on the notion that Bayesian average increase the robustness of the model and reduces the pitfall of overtraining or being overly sensitive to a specific parameter setting.

The binary form of the pMax was used in the iteration as shown in the Fig. 3. The notations used in the procedure were: \( P_n \), the pMax matrix for alpha value \( n \); \( B_n \), the binary form of \( P_n \); \( A_{0.1} \ldots m \), the average of \( B_{0.1} \) to \( B_m \) (the average of \( B_{0.1}, B_{0.2}, B_{0.3}, \ldots, B_m \)). Any value greater than 0 in the average matrix \( A_{0.1} \ldots m \) was treated as a connection. The iteration always starts with \( m \) which is the max alpha that we use in our iteration equals 0.9.

Fig. 6. Results from simulation 1, 10 node network: to compare our results with the previous work (Smith et al., 2011) we applied the algorithm on individual subjects. Panel (a) shows the C-sensitivity for connection/edges detected. (The connection strengths for every connection were assigned as \(-\log(\alpha)\) where alpha is the lowest value at which the connection first appears (Smith et al., 2011). Then we calculated the number of true connection whose strength was greater than the 95% percentile of the false positive connection strength. This number was divided by the total number of true connections. This fraction is defined as c-sensitivity in Smith et al. (2011).). The white dot in the figure shows the mean of the distribution which is above 0.9 i.e. 90%. This finding replicates prior findings by Smith et al. (2011) and panel (b) shows the probability of correct direction. The mean as shown in the figure is around 0.5 which is 50%.

The results from simulation 2 are shown in Fig. 7. Panel (a) shows the estimated model for Data set 1, contemporaneous (b) Estimated model for data set 2, contemporaneous and lagged, and (c) Estimated model for data set 3, lagged only. Structure links are indicated with blue or red arrows. Blue arrows indicate correct direction, and red arrows indicate incorrect direction.
To illustrate how the iterations work let us take an example. The iteration begins with $m = 0.9$.

Step 1: $T = m - 0.05 = 0.85$ and $C = 0$, $A_{0.1-0.9}$ is calculated.
Step 2: Check if the structure (connections) in $A_{0.1-0.9}$ is same as $B_T = B_{0.85}$.
Step 3: (A) If the structure is same find the lowest $n$ for which the structure of $B_n$ is still same as the average. The structure and the directions of the connection for this $B_n$ are treated as the true ones. There is a range of alpha levels where the structure remains constant. So the lowest alpha value where the structure remains consistent is considered to be the most stable for the structure and the directions.

(B) If the structure is not the same, $C = C + 1$;
   (i) If $C = 1, T = T - 0.05 = 0.85 - 0.05 = 0.8$ and go to step 2.
   (ii) If $C = 2, m = m - 0.1 = 0.9 - 0.1 = 0.8$ and go to step 1.

To compare the results with Smith et al. (2011), we ran $pc()$ on individual subjects for logarithmically spaced alpha between 0.01 and 1 on data set 2. We assigned connection strengths for all the connections as $-\log(\alpha)$ where alpha is the lowest value at which the connection first appears (Smith et al., 2011). Then we calculated the number of true connection whose strength was greater than the 95% percentile of the false positive connection strength. This number was divided by the total number of true connections. This fraction is defined as c-sensitivity in Smith et al. (2011). C-sensitivity was calculated for each individual subject separately and averaged across the subjects. For the directionality, we found the most consistent alpha for each subject and found the probability of the true connections to have true directions.

For simulation 2, the averaged maximum correlation was considered as input for the contemporaneous plus lagged relation model and lagged relation model to generate the pMax matrix. For the contemporaneous network, the input was average correlation as was in Simulation 1 from Smith et al. (2011). As explained earlier, the binarized pMax was generated and rest of the iteration was carried out.

For the empirical data, we used the three networks namely default network, fronto-parietal network and cingulo-opercular network, which we found with modularity analysis (see Fig. 8). The structural data contains the information about the number of fibers connecting two nodes. This data was combined for each connection across all subjects and recoded as binary (connected versus non-connected) as follows. If the connection appears in more than 50% of the subject, then the connection was assigned a 1 or else 0. This binarized structural data was treated as the gold standard (baseline). For each network, the correlation coefficients for the functional data were averaged for the 8 subjects. The connections from this functional connectivity and PC algorithm were then compared for 100 different thresholds to the baseline and the true positive rate (TPR) and false positive rate (FPR) were found.

Results

Simulation 1

As mentioned earlier, eight sets of simulations were used from the Smith et al. (2011) study. Fig. 5a shows the structure and direction of the information flow recovered by the algorithm for the data set with 5 nodes. The algorithm was able to recover all the connection and directions accurately. Similarly the structure and the causality for 10 nodes data and 50 nodes data were accurately recovered. Fig. 5b shows the result for the data set with 10 nodes. The algorithm even worked excellently for varied session duration (data sets 4, 5 and 6). For the cyclic connection, the structure was accurately recovered but the direction indicated in Fig. 5c in red was identified incorrectly. Fig. 5d shows the result from the algorithm for backward connection. The ground truth by Smith et al. (2011) for the backward connection is shown in SI Fig. 1.

As explained earlier, we calculated the c-sensitivity for each subject. Fig. 6a shows the distribution of c-sensitivity for all the subjects. The white dot in the figure shows the mean of the distribution which is above 0.9 i.e. 90%. This finding replicates prior findings by Smith et al. (2011). Fig. 6b shows the probability distribution for the true direction and the mean as shown in the figure is around 0.5 which is 50%.

Simulation 2

For Simulation 2, the algorithm was successful in finding the structure for all three data sets (Figs. 7a, 7b and 7c). The algorithm was also somewhat successful at identifying directionality. As shown in Fig. 7a, for the contemporaneous data, four out of eleven directions were incorrect. Out of the four two were bi-directional meaning the algorithm was not able to decide how to orient these connections. As shown in Fig. 7b, for the mixture of contemporaneous and lagged data and only lagged data, five out eleven connections were oriented incorrectly.

Empirical data

Modularity results

As mentioned in Modularity analysis section, modularity analysis was applied to our empirical structural matrix to detect networks within the structural data. This data driven approach splits the whole brain into 5 different networks namely default network, fronto-parietal network cingulo-opercular network, visual network,
and motor network. We use default mode network, fronto-parietal network and cingulo-opercular network as shown in Fig. 8 to examine how functionally deduced connections with our algorithm and traditional correlation methods relate to the structural connections within these identified networks.

Comparison
As noted above, here we compared the structure based on the Pearson’s correlation and the structure that we obtain from pc() with 3 networks identified with community detection using the structural data (Fair et al., 2009; Newman, 2006). The three networks included the default network (pink), the cingulo-opercular network (red) and the fronto-parietal network (green).

Fig. 9 shows the radar graph indicating the difference between the true positive rate (TPR) and false positive rate (FPR) for the two methods. The value on the circumference of the circle indicates the different levels of thresholds. The values on the radar axis (vertical axis) indicate the difference. As one moves towards the edge of the radar graphs the better the functional connectivity agrees the structural connectivity baseline. For all three networks we can see that PC algorithm was more accurate than the straight correlation. We also conducted a t-test between the PC algorithm and straight correlation for the difference of TPR and FPR. We found that for all the three networks the p-value was <0.0001.

Discussion
In recent years, resting-state functional connectivity MRI (rs-fcMRI) has become popular tool that measures temporally correlated low-frequency BOLD signals in subjects at rest (Biswal et al., 1995). This measurement of functional relatedness has offered significant insight into brain organization. However, most investigators examine the functional relatedness of regions using Pearson’s correlation coefficient, which has the advantage of ease of application, but is not able to differentiate between direct and indirect influences or identify the direction of information flow (i.e. directed functional connectivity). In this report we examine the ability of the PC algorithm, based on conditional independence to elucidate these aspects of connectivity on two simulated data sets and an empirical data set.

We find that, as previously reported by Smith et al. (2011), based on individual subject data, the approach yields relatively strong results with regard to identifying direct versus indirect connectivity; however, the algorithm does poorly with regard to directionality on individual subject. In contrast, applying the algorithm to averaged connection matrices yields strong results for both indirect/direct connections and directionality for some simulated models (Smith et al., 2011). The method provides modest results with regard to these outcomes for the other models (Simulation 2 (Gates and Molenaar, 2012)) as well. In the latter case, the simulated data contains lagged information in two of the data sets, which reduces the efficacy somewhat. Note, probabilistic inference of direction of influence has certain inherent limitations. Of the three elemental structures, only one of the V-structure can be inferred unambiguously. In many applications where Bayes network can be successfully inferred, the V-structures over all the three variable combinations strongly constraints the directions of the influence amongst all the variables. In both of our simulations, the underlying structures had links that cannot be unambiguously resolved; however, we thought it is important to begin our inquiry with data structures that have been examined in prior reports (Gates and Molenaar, 2012; Ramsey et al., 2011; Smith et al., 2011). Importantly, using empirical data to compare the correlation based method and the PC algorithm to the structural data; we see that the PC algorithm performs better than the traditional correlation methods.

We note that the work by Smith et al. (2011) was followed up by at least two studies beside ours. First, Ramsey et al. (2011) applied two algorithms IMaGES (modification of Greedy Equivalence Search) and LOFS (modification of Linear, Non-Gaussian, Acyclic Causal Model) on the Smith et al. (2011) data. Looking at the results from data set with 10 nodes, the two methods correctly identified nearly all the connections and 90% of the directionality on group of subjects. They used 10 subjects out of the 50 subjects with replacement to calculate the connection and orientation precision. Notably, when using the algorithm on single subjects, the algorithm correctly identified 90% of the connections right but only 63% of the causal directions correct. Second, (Gates and Molenaar, 2012) used Group Iterative Multiple Model Estimation (GIMME) which is an extension of unified structural equation modeling (uSEM) (Gates et al., 2010). Individual and group level analyses were carried out with this algorithm re-examining the 10 node data set from Smith et al. (2011). For Smith et al. (2011) the 10 node data set, the individual-level uSEM obtained a c-sensitivity of 90% but only detected 50% of the directions. For the GIMME analysis (50 subjects), all of the connections were recovered and 90% of the directions were accurate for Smith’s 10 node data set. The algorithm becomes highly complex for larger number of nodes and may be difficult to apply to large networks even for group level analysis. It will be important for long-term that the methods applied to these types of data

![Fig. 9. Radar graph: the graph shows the difference between True positive rate and the False positive rate when comparing PC algorithm and straight correlation with the structural data for each network (a) default mode network, (b) cingulo-opercular network and (c) fronto parietal network.](image-url)
are able to handle large datasets, as whole brain systems could include large number of nodes even more than 50. Thus, while these methods all show some promise, the results all highlight the need for further methods development. Furthermore, all of these tests examined thus far have looked explicitly at the same single simulated dataset, which may not reflect all of the complexities of empirical data. The work in this report begins to fill some of this void.

Threshold related problems and our solution

One parameter that has been difficult to deal with when using correlation based measures is what threshold to signify connected versus non connected nodes. Although it has been shown that thresholding is a necessary step to remove spurious connections while generating a graph (Rubinov and Sporns, 2010), the choice of threshold is a critical decision point in analytical processes. A choice of R approaching 1.0 will generate very sparse graphs, with a limited number of edges (i.e., few connections). On the other hand a choice of R approaching 0.0 will generate densely connected graphs. The PC algorithm has a similar problem; alpha is the tuning parameter for PC algorithm (Kalisch and Buhlmann, 2007). The structure, number and the orientation of the edges depend on the alpha level. In other words, as with correlation based methods, network changes depend upon the alpha chosen.

As an example we applied three differing thresholds using the 10 node model (data set 2) from Smith et al. (2011) — each of which provides a unique result. At one alpha the true network structure is revealed, and at the others the structure is lost (see Fig. 10). Because of these discrepancies we attempted to generate an algorithm that could identify the structure of the system independent of a specific alpha, but rather a system that would rely on the most consistent level of alpha. Our method is inspired by ideas of Bayesian smoothing where the models are smoothed across different parameter settings using a prior. In our case, there is no information prior hence a uniform prior implies averaging the results from all the settings. The current report using both simulated and empirical data suggest that this approach provides consistent and accurate findings with this regard.

Comparing the PC algorithm implementation in the current report with the Smith et al. (2011) implementation

In Smith et al. (2011), 38 different methods potentially capable of determining true network structure and in some instances directionality of information flow in BOLD fMRI data were examined. Partial correlation, ICOV and various Bayes method performed well in finding the true network structure but none of the methods were capable of determining the direction of information flow when using single subject data. In the current manuscript we focused on one of the Bayes methods tested by Smith et al. (2011) — the PC algorithm (Sprites et al., 2000). The PC algorithm functions by building a Bayesian Network from the data based on conditional independence tests on subsets of three variables. Although the algorithm used here produced only a slightly better result than Smith et al. (2011) in its ability to detect the true network structure in individual subject data, when data was averaged across participants, the results were largely superior. In the case of averaged data, the algorithm did better job at detecting directionality, as well. The analysis on the individual level is likely more prone to inherent noise, whereby using group averaged matrices likely improved the signal to noise ratio, thus allowing for the superior result. This is important support for the PC algorithm because for many research questions, averaged data are sufficient.

The differences in the varying results are also likely secondary to the two program's (R and TetrA) implementation of the PC algorithm. The PC algorithms used in the two papers were slightly unique. Smith et al. (2011) used the tetrad software, while here we used an algorithm implemented in R. While the implementations are largely similar, there are some differences. These differences appear to be nested in the implementation of the conditional independence tests (or Meek's rules http://uai.sis.pitt.edu/papers/95/p403-meek.pdf) as shown in SI Fig. 2 and the post-processing of the Bayes network. The PC algorithm in R uses the orientation rules described in the book (Sprites et al., 2000, p 85). The algorithm uses some but not all the four Meek's rule for orientation of pattern where as the pc algorithm in tetrA uses all four Meek's rule (http://www.phil.cmu.edu/projects/tetrad_download/files/manual.pdf). These rules are illustrated in SI Fig. 2 and are also explained below.

| R (Rule) 1: if A → B–C and if A and C are not adjacent then orient B → C. |
| R2: if A → B ← C and A–C, orient A → C. |
| R3: if A → B ↔ C, A is not adjacent to C and A–D–C, then orient D → B. |
| R4: if A ↔ B ↔ C, A is not adjacent to C and A–D–C, then orient D → A. |

Also tetrad stops its processing after these rules are implemented. With the R implementation one can include an option which runs the conditional independence test along with 100 different combinations for an ambiguous direction. The orientation of the edge is overwritten at later point if a new orientation is found.

Backward and cyclic connections

Of note, when one considers feedforward and feedback connections together for any given circuit in the simulation, the method performs poorly on both respects. While simulating feedback connections for fMRI data is not straightforward (Smith et al., 2011), the findings based on how they were modeled in the data by Smith et al. (2011) are potentially informative and an important consideration. The large majority of links in the brain contain both feedforward and backward connections. In the neocortex, most of the feedforward connections arise from supragranular layers, i.e. layers II and III, whereas feedback connections arises from the infragranular layers, i.e. layer

Fig. 10. Different thresholds provide different network structures. The implementation of our algorithm (Fig. 3) is inspired by ideas of Bayesian smoothing and accounts for this problem where the models are smoothed across different parameter settings using a prior. In our case, there is no information prior hence a uniform prior implies averaging the results from all the settings. (a) Network structure at 0.48 alpha value (b) network structure at 0.93 alpha value.
V and VL. Another factor to consider is inhibitory and excitatory connections, which can also be difficult to model, but that change considerably in the life span and are likely to reduce the accuracy of methods aimed at identifying direct and indirect links in fMRI data. In addition, other aspects of the model, such as the presence or absence of cyclic connections add complexity to the discovery (Friston, 2011). In other words, the relationship of feedforward and feedback connections, inhibitory and excitatory connections, and the presence or absence of cyclic connections in the brain is significantly complex and unlikely to be fully captured by any given model considering the relatively sluggish fMRI BOLD response. It is likely that current models (for full list see Smith et al., 2011) aimed at identifying true connection structure and directionality are unable to incorporate all these true biological dynamics.

Learning the links of influence between variables in a Bayesian network is a challenging problem with combinatorial complexity. The PC algorithm simplifies the task by breaking up the problem into independent subtasks. At the most elemental level, the subtasks involves testing conditional independence between three variables. Among the three possible directed influence structures amongst three variables, one of them (often referred to as the V structure) is unambiguous. The ambiguous links of influence are subsequently resolved by finding an explanation that is consistent with results from all the other three variable subtasks. In certain cases, there may not be a consistent explanation or the explanation may not be sufficient to resolve all ambiguities in the network. This is the trade-offs made by PC algorithm to simplify the structure learning problem. So the accuracy of the algorithm depends on the underlying true structure that generated the data and hence is dependent on the application. Thus, while apparently better than traditional correlation analyses, the PC algorithm may not perform optimally in “real world” systems (Friston, 2011). Our examination of empirical structural and functional data using the PC algorithm supports this notion. We note that Dynamic Causal Modeling (DCM) was developed with some of the above noted limitations in mind and might be used to improve on such findings — for details see Friston (2011).

Homogeneity assumption

The simulated data network examples used here were homogeneous across individuals. Testing novel methods against homogenous data sets reflects the current standard. However, emerging evidence suggests that such simulations may not be entirely representative of data sets found in empirical studies because heterogeneity likely exists across individual brain maps (Fair et al., 2012) see also (Hillary et al., 2011; Kherif et al., 2003, 2009; Seghier et al., 2008). In the presence of heterogeneity, empirical (Miller and Van Horn, 2007; Miller et al., 2002) and simulated (Molenaar et al., 2002) data examples have frequently pointed out that traditional group-level analysis results in findings that fail to describe any individual comprising the group. The degree to which the aggregated result differs from individual results relates to the degree of heterogeneity found across individuals (Gates and Molenaar, 2012). Thus, heterogeneity may present a problem only in extreme cases where individuals vary greatly. Since aggregation in some form has been shown to pick out signal from noise (e.g., Ramsey et al., 2011), the benefit may outweigh the negatives in some cases. More work is needed to ascertain the degree of heterogeneity that exists in specific populations and how this may affect the reliability of methods, such as the present one, which aggregates across individuals using averaging.

Empirical data

As noted above, while the simulations by Smith et al. (2011) and the additional simulations (Gates and Molenaar, 2012) included here are a good start, there are limitations to these simulations. One of the problems with these models is that they are not small-world i.e., clustering coefficient = 0. This reduces their biological plausibility. Hence to test the procedure in a more biological plausible context, in the present report we attempted to generate an empirical test of our method compared to the traditional correlation analyses by examining how close the methods relate to the underlying structural connectivity based on high resolution tractography. While we recognize that high resolution tractography has its own limitations (Dell'acqua and Catani, 2012; Masutani et al., 2003), we believe it likely reflects an estimate of the underlying structural matrix and should serve as a good baseline to compare methods. In this instance, the method applied to the fMRI data best able to identify the true structure of the matrix should recapitulate the average matrix and network structure of structural connectivity data.

Despite limitations in the method’s ability to clarify structure in data that includes feedforward and feedback connections, the PC algorithm outperformed the traditional correlation based method. However, while the Bayesian networks appear to outperform traditional correlations, in identifying structure neither are significantly strong in these biologically plausible scenarios as explained above. This lack of correspondence is potentially related to several factors. The imprecise nature of the methods as noted in our simulations is likely leading to reduce efficacy in identifying the true structure of the matrix. However, it is possible that the low performance obtained by both methods is at least partially reflective of the imprecise nature of the baseline, or “gold standard,” used here, being high resolution DWI.

Nonetheless, in all instances it appears that the PC algorithm, as implemented here, outperformed traditional correlation methods suggesting that it is modestly superior at reflecting the true underlying connectivity structure.

Conclusion

In conclusion, these findings reveal that 1) probabilistic graphical model learning methods such as the PC algorithm may be superior at matching functional connectivity matrices to structural connectivity, 2) in the simplest, non-biological, cases these methods can reveal both structure and directionality, but 3) when considering empirical biological data, or more biologically plausible simulations, such as feed-forward and backward connections (a phenomena quite ubiquitous in the brain), both methods perform poorly at identifying directionality.

We believe these results suggest a potentially alternative way to examine connectivity using BOLD fMRI data; however this method cannot be applied to find the direction of information flow. Future work will need to build on approaches, as examined here, that provide a baseline for comparisons purposes and additional neuronal and BOLD models that better capture various biological phenomena.

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.neuroimage.2013.02.054.

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Conflict of interest

All authors report no biomedical financial interests or potential conflicts of interest.